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DYNAMIC MODELING OF A LINEAR QUADRATIC REGULATOR BASED OPTIMAL DIRECT CURRENT MOTOR FOR IMPROVED PERFORMANCE

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ABSTRACT

The paper herein presents the dynamic modelling of a linear quadratic regulator based optimal direct current (DC) motor controller for improved performance. This study investigates the optimal use of a controller in the control of DC motor speed. This work also carried out a comparison of time response specification of the Linear Quadratic Regulator (LQR) for a speed control of a separately excited DC motor. This paper investigates the appropriate control strategy that delivers a better performance with respect to DC motor's speed reliability. The control method to be implemented in this paper is the LQR which is a state space controller and the model of a direct current motor is presented in state space form. MATLAB/SIMULINK software program was used to simulate the steady state and transient mathematical models of the machines. The test for both controllability and observability were carried out in the rank of matrix two. When the LQR was simulated, it was observed from the plots of speed against time at open loop with unity value of 1Volt and 1Ohm, that the DC motor system with the LQR control stabilizes faster at the speed of 1rad/secand after about 1seconds.

Keywords: *DC motor, PID controller, LQR controller, modelling, speed control, state space analysis*.

I. INTRODUCTION

It is a significant reality to know that the DC (direct current) machines have been seen as the most widespread transponders devices; their distinguished importance/advantage is that the voltage and current (Volt-Ampere) or the characteristics of the speed torque of these machines are so much flexible and adjustable and are subjected to operate in both steady state and dynamic operations. Electrical machines are highly essential in our modern society and our day-day lives (Slemon*et al*, 1982). The direct current machines are used to produce electrical energy whether it is required in the usable heat form (thermal application), mechanical power form (electrical motor in industrial application), illuminating form (lighting system) or conversion system. The immediate current machines likewise perform or work as a motor in a direct current form and as a generator, the exhibition and activity of a DC machine can change starting with one mode or structure then onto the next consequently and alternately or the other path round. It is most advantageous to take a gander at the different working of the DC generator in light of the fact that the enthusiasm of this generator is the terminal voltage with burden and speed variety, while in a DC engine, the distinctive highlights are the connection or association existing apart from everything else of power (torque) and (speed) and the modification or compliance to different types of mechanical burden (Sen et al, 1989). Another very important or imperative feature of a DC machine is due to the potentiality or capability of developing an extensive number or collection of torque/speed characteristic, its economical speed control (Rizzoni*et al*, 1996), and forever-multiplying intricacy or enlargement of industrial processes that needs larger flexibility from electrical machines in terms of special characteristics (Ryff*et al*, 1987). Direct current motor been long established are Fractional-Kilowatt dc motors that have higher efficiencies and are minute in frame and sizes than their AC competitors. However, the DC motors have some natural disadvantages too concerned with commutation and magnetic phenomena. Traditional or ordinary commutation in a small machine has approximate small sections, giving rise to emf and torque ripples. Brushes are responsible for voltage drop, friction wear and radio interference. Primarily, large motors can be used as

control devices (tacho generators) for speed sensing and servomotor for positioning and tracking, also in machine tools. Printing press, textile mills, pumps, hoist, and conveyors fans (Slemon*et al*, 1982).B. T. Polyak (2001),applied linear quadratic regulators (LQR) to study robustness of a system for a desired output in terms of reliability. Systems affected by random bounded nonlinear uncertainty were considered so that classical optimization methods based on linear matrix inequalities cannot be used without conservatism. The approach used was a blend of randomization techniques for the uncertainty along with convex optimization for the controller parameters. J. M. Dores Costa (2010). Presented the Design of Linear Quadratic Regulators for Quasi-Resonant DC –DC Converters", Costa in his work, proposed that a law called linear feedback most often are responsible for increasing the dynamic performance of the transducer (converter) in small signal forms. He suggested that this could be achieved by quasi-resonant converter which involves a state space method. The author stated the research can be also be carried out when a vector quantity with gain margin is placed t help get the closed-loop very nearer to the intending locations. Instead, he used optimal control techniques, together with those small-signal models, to design a robust voltage controller for QR converters that overcomes some uncertainties that may corrupt the results obtained by classical design methods. Anders Rantzer (2000). Carried out his research work titled: "Piecewise Linear Quadratic Optimal Control", the use of piecewise quadratic cost functions was extended from stability analysis of piecewise linear systems to performance analysis and optimal control. Lower bounds on the optimal control cost were obtained by semi-definite programming based on the Bellman inequality. This also gave an approximation to the optimal control law. A compact matrix notation was also introduced to support the calculations and it was proven that the framework of piecewise linear systems could be used to analyze smooth nonlinear dynamics with arbitrary accuracy. John Swigart (2010) Proposed a control measure inform of algorithms for handling different controller problems, where individual subsystems were connected over a network. Their work was focused on the simplest information structure, consisting of two interconnected linear systems, and they constructed the optimal controller subject to a decentralization constraint via a spectral factorization approach. In their research, they proposed an explicit method of analysis called state-space method which helps in the optimization of the control process, characterized its order, and show that its states are those of a particular optimal estimator. Joel Douglas (1994). By applying the over bounding method of Petersen and Hollot in 1994 derived the controller (LQR).This controller is optimal and robust in handling real life parameters. The Algebraic reccati equation is a very powerful means handling the control methods of DC machines. This controller has the same guaranteed robustness properties as standard linear quadratic designs for known systems. It is proven that when applied to a structural system, the controller achieves its robustness by minimizing the potential energy of uncertain stiffness elements, and minimizing the rate of dissipation of the uncertain damping elements. Michael Safonov (1976) had explained in their work that multi-loop linear-quadratic state-feedback (LQSF) regulators demonstrated robustness against a variety of large dynamical, time-varying, and nonlinear variations in open-loop dynamics. The results were interpreted in terms of the classical concepts of gain and phase margin, thus strengthening the link between classical and modern feedback theory.
Andrew (2013), He proposed that linear quadratic regulator problem can be solved when the state of the system under consideration is known. They concluded that the optimal controller is linear and can be computed from a generalization of the classical Riccati differential equation.

II. MATERIALS AND METHODS

Materials: The materials used in the actualization of this work are as follows:

- A Direct Current (DC) Motor Model
- MATLAB/SIMULINK R2013b software
- A HP Window10 laptop for implementation
- Journals

Methods of Modelling the System

The system under consideration is a DC motor. The schematic diagram of a DC motor is given in figure 1.

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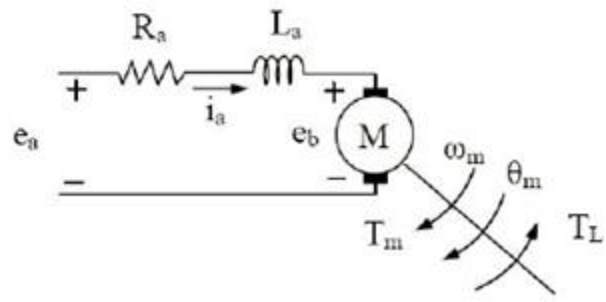

Figure 1: Schematic Diagram of a DC Motor (Katsuhiko Ogata, 2002)

Dynamic equations of a dc motor:

- $E_a = i_a R_a + L_a \frac{di_a}{dt} + E_b$ (1)
- $E_b = K_b \omega_m = K_b \frac{d\theta}{dt}$ (2)
- $E_a = i_a R_a + L_a \frac{di_a}{dt} + K_b \frac{d\theta}{dt}$ (3)
- $T_m = J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt}$ (4)
- $T_m = K_t i_a$ (5)
- $K_t i_a = J_m \frac{d^2\theta}{dt^2} + B_m \frac{d\theta}{dt}$ (6)

Equations 1 to 6 shows the dynamic equations of the DC motor.

There are several methods or ways to describe or model a system which include
i Transfer function and
ii State space representation among others.

The control to be implemented in this paper is a Linear Quadratic Regulator, which is a state space controller. Hence, the model of the direct current motor will be presented in State space form. The general state-space representation of a linear system with *m* inputs, *p* outputs and *n* state variables is written as,

- $\dot{x}(t) = Ax(t) + Bu(t)$(7)
- $y(t) = Cx(t) + Du(t)$ (8)
-

$$\frac{d}{dt}\begin{bmatrix}\omega_m \\ i\end{bmatrix} = \begin{bmatrix}-\frac{B_m}{J_m} & \frac{K_t}{J_m} \\ -\frac{K_b}{L_a} & -\frac{R_a}{L_a}\end{bmatrix}\begin{bmatrix}\omega_m \\ i\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{L_a}\end{bmatrix}E_a \quad (9)$$

$$y = [1 \;\; 0]\begin{bmatrix}\omega_m \\ i\end{bmatrix} + [0]E_a \quad (10)$$

Equation (9) and (10) make up the state-space representation of the DC motor system. There are two state variables x_1, x_2,correspondingtoω_m, i_a respectively, and this means that the system is second order with two poles. The input is the voltage E_a and the output is the angular velocity ω_m. So the DC motor has been put in the form given in equation (7). This is an example of a single-input single-output (SISO) system and hence there is no D matrix. The system was simulated using the following parameters (Katsuhiko Ogata, 2002).

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Table 1: Parameters for LQR control simulation

Parameter	Value and Unit
E_a	12 volts
J_m	0.01kg.m^2
B_m	0.00003kg.m^2/s
K_t	0.023Nm/A
K_b	0.023V/rad/s
R_a	1Ω
L_a	0.5H

(Nguyen, 2006).

Substituting the parameters in equations (9) and (10), the model of the SEDM is given as,

$$\frac{d}{dt}\begin{bmatrix}\omega_m\\ i\end{bmatrix} = \begin{bmatrix}-0.003 & 2.3\\ -0.046 & -2\end{bmatrix}\begin{bmatrix}\omega_m\\ i\end{bmatrix} + \begin{bmatrix}0\\ 2\end{bmatrix}E_a \quad (11)$$

$$y = [1\ \ 0]\begin{bmatrix}\omega_m\\ i\end{bmatrix} + [0]E_a \quad (12)$$

Open loop simulation

The SEDM system was then simulated in **Matlab/Simulink** to analyse the response of the system without any form of control. The block diagram in figure 3.2 describes how the model was built in Simulink.

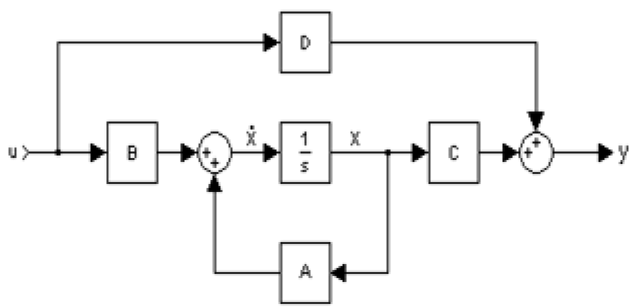

Figure 2: Block diagram of state space model

III. CONTROL DESIGN

The control technique proposed for the SEDM system is the Linear Quadratic Regulator. Given the system represented in equation (7), where the pair (A,B) is stabilizable and the pair (A,C) is observable, the LQR problem is to determine the optimal gain K, such that the control law,

$$u = -Kx \quad (13)$$

Minimizes a performance index,

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \quad (14)$$

Where $Q \geq 0$ is a diagonal matrix which is the weight on the state vector, $R > 0$ is the weight on the control action, u. (Katsuhiko Ogata, 2002).

It can be shown that the optimal gain is given by,

$$K = R^{-1}B^T P \quad (15)$$

Where P is the solution to the algebraic Riccati equation,

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (16)$$

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Test for controllability and observability

In order to ensure that there exists an optimal gain *K* which can minimize the performance index *J*, the pair (A, B) must be stabilizable and the pair (A, C) must be observable. To check for these conditions, the controllability and observability matrices have to be computed and analysed as follows:

The controllability matrix of a system represented by equation (7) is given as,

$Ctrb = [B \;\; AB \;\; A^2B \;\; A^3B \ldots A^{n-1}B\,]$(17)

In the case of the SEDM, *n* = 2, hence, the controllability matrix is,

$Ctrb = [B \;\; AB]$(18)

$$B = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \text{and} AB = \begin{bmatrix} -0.003 & 2.3 \\ -0.046 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 4.6 \\ -4.6 \end{bmatrix}$$

Thus, $Ctrb = [B \;\; AB] = \begin{bmatrix} 0 & 4.6 \\ 2 & -4.6 \end{bmatrix}$which has a full rank of 2. Hence, the pair (A, B) is controllable and hence stabilizable.

Similarly, the Observability matrix of a system represented by equation (7) is given as,

$$Obsv = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

In the case of the SEDM, *n* = 2, hence, the observability matrix is,

$$Obsv = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$C = [1 \;\; 0]$and

$$CA = [1 \;\; 0] \begin{bmatrix} -0.003 & 2.3 \\ -0.046 & -2 \end{bmatrix} = [-0.003 \;\; 2.3]$$

Thus, $Obsv = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -0.003 & 2.3 \end{bmatrix}$

which has a full rank of 2. Hence, the pair (A, C) is observable.

Thus, it is certain that we can find an optimal gain *K* which can stabilize the DC motor using LQR.

Selection of Q and R matrices

The selection of Q and R was done through a trial and error process, which is the standard way of selecting Q and R. As stated earlier, the Q matrix is an n x n matrix and is a diagonal matrix which represents the weight on the state variables *x*. Increasing Q means that the state variables must be small, in order to minimize the cost function J while reducing Q means that the state variables must be large in order to minimize J. The matrix R is an *m* x *m* matrix and is the weight on the control action *u*. Selecting large R means that the control input (input voltage in this case) must be small to minimize J while selecting small R values means that the control input must be large to minimize J. There is a standard selection of Q and R which is given as $Q = C'C$and$R = 1$. This gives a selection of weighting matrices as,

$$C'C = \begin{bmatrix} 1 \\ 0 \end{bmatrix} [1 \;\; 0] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{and } R = 1.$$

The solution to the Riccati equation given in (3.22) is *P*, and can be obtained mathematically using the Hamiltonian matrix (Katsuhiko Ogata, 2002) and very complex arithmetic. But to simplify the process, the matlab*care* function was simply used to obtain *P*. The *'care'* stands for Continuous Algebraic Riccati Equation, and it is used to find *P*, the solution of the Riccati equation.

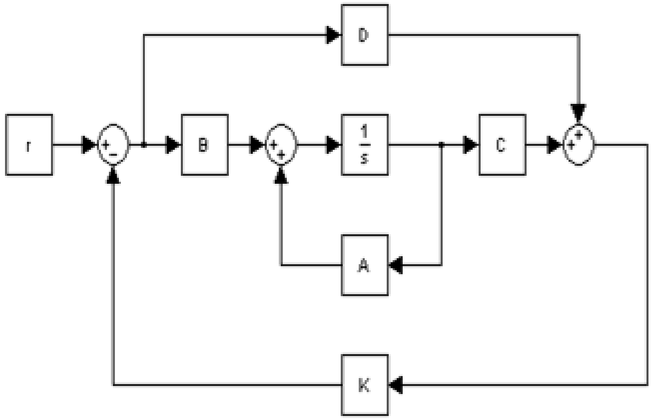

Figure 3: LQR control scheme for the DC motor

IV. RESULTS AND DISCUSSION

The setup shown in figure 3.2 was implemented in Matlab/Simulink. An open-loop step response of the system was obtained using a step input *u* of IV, and measuring the output *y*. The result obtained is shown in figure 4.

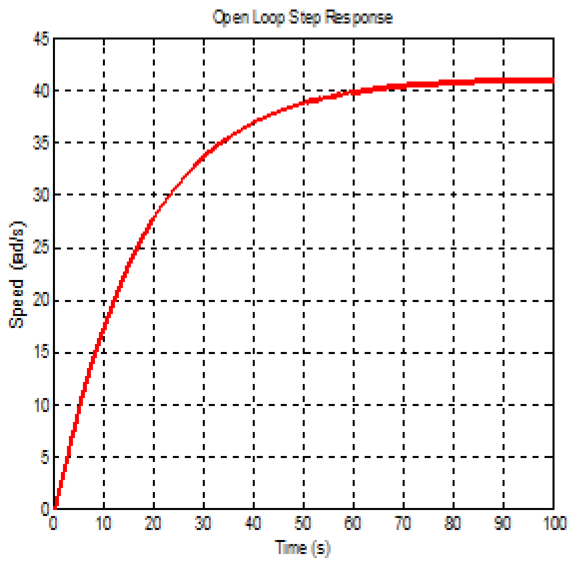

Figure 4: Open-loop step response of the system

It can be observed from figure 4 that the DC motor system is stable in open loop. This is because for a unit voltage (1V) input to the system, the speed increases continuously as time increases, until it stabilizes at a speed of 41rad/s after about 80 seconds. This stability is further confirmed by the poles of the motor system. The poles of the system are the eigenvalues of the A matrix in the state space model. These poles were computed in matlab using the '**poles**' command and found to be -0.0575, -1.9455. Since both poles are in the left half plane, the DC motor system is stable.

However, even though the motor system is considered stable, the time domain parameters such as rise time, time constant, settling time etc. are not satisfactory. For instance, the settling time is approximately 80 seconds, which is too long. The time constant which is the time taken to reach 60 percent of its final value can also be seen to be around 17secs (corresponding to 25.83rad/s which is 63% of 41rad/s). This is also very large. Again, the rise time from 0% to 90% of the final value is seen to be 40 seconds (corresponding to 36.9rad/s which is 90% of 41rad/s). This value too is also very large.

In order to improve these parameters and generally increase the speed of the overall motor system, a Linear Quadratic Regulator (LQR) is proposed to control the SEDM and make it maintain any speed (measured in rad/s) which is desired by the user. The design of the LQR controller is discussed in the next section.

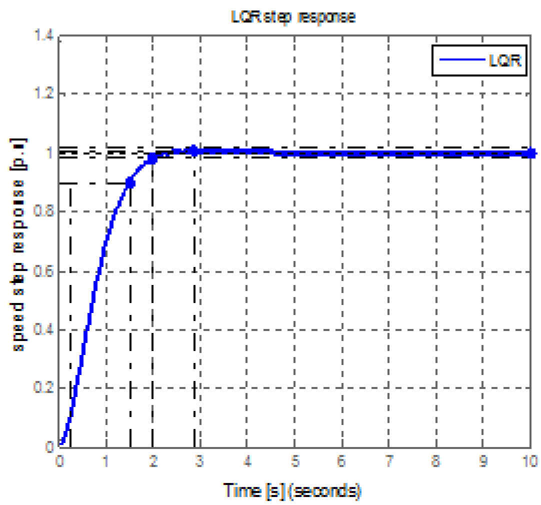

Figure 5: Speed output with LQR and typical Q and R values

It can be observed from the figure 4.10 that the SEDM system with the LQR control stabilizes much faster than the open loop response. This agrees with the faster (larger) closed loop pole values obtained. There is a slight overshoot and very minor oscillations due to the presence of a complex part in the pole pair. It can also be observed that there is no steady state error and the LQR controller is able to achieve reference tracking.

It should be noted that the typical selection of $Q = C^TC$ and R = 1 means equal weighting or equal penalization on both the state variables and the control action.

Table 2: Speed variation of LQR for J (moment of inertia)

System J	Rise Time(Seconds)	Settling Time(Seconds)	Steady State	Overshoot (%)	Peak Amplitude
LQR	1.260	1.990	1.000	0.525	1.000

Tables 2 shows the summary results of LQR obtained from the plots of the figure below for the variation of moment of inertia (J = 0.01kg.m^2, 0.5J and 2J).

Figure 6 shows the graph of LQR step responses for the variation of moment of inertia as computed using matlab

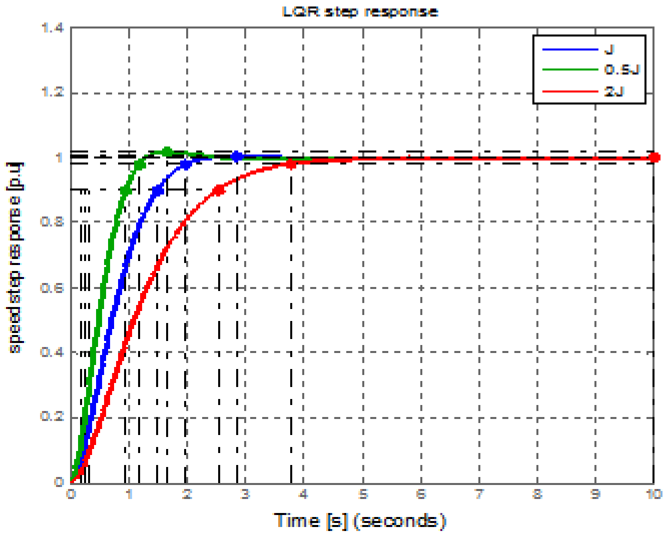

Figure 6: Graph of LQR step response for variation of moment of inertia

Figure 6shows the variation of moment of inertia of the LQR in terms of rise time, settling time and steady state for J, 0.5J and 2J respectively.

In summary, the LQR controller is able to stabilize the system to track a reference point as shown in figure 5. The control resulted in a small time constant, rise time and settling time, compared to the open loop and PID system. There is also no steady state error and the frequency response analysis has shown that the system has a wide stability range.

V. CONCLUSION

In this paper the optimal LQR controller for DC motor speed regulation has considered. The performance of the controllers is validated through simulations of the different models. Simulation results are presented and discussed. From the results it can be seen that the linear quadratic regulator controller realizes a good dynamic behavior of the DC motor with a rapid rise time, settling time, little or no overshoot, and zero steady state error under nominal condition. But the result by the linear quadratic regulator technique shows clearly that the LQR technique gives better performances than other conventional controllers. Furthermore, the simulation results obtained shows that the LQR gives very good time response specifications in terms of percentage overshoot rise time, settling time steady state, peak amplitude etc.

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